Question 1

In a very long cylindrical and homogeneous axon an action potential propagates.
The x-axis is taken to be the symmetry axis.

a. At one instant in time the peak of the travelling action potential reaches x=0.
At that moment the region from x=-2b to x=+b is depolarized. Assume that the intra-cellular potential from x=-2b to x=0 and from x=0 to x=+b varies linearly.
The intra-cellular conductivity is σ_i the extra-cellular conductivity is σ_o.
Find an expression for the field-potential recorded at (x_0, y_0).
Help: for a homogeneous medium with conductivity σ_o the potential at a distance r from a current source i - the solution for the space integral shown in the powerpoint slides- is:

$$v(r) = \frac{i}{4\pi\sigma_o}\frac{1}{r}$$

b. When the axon depolarizes a constant current dipole **p** pointing to the right sweeps along the axon with constant velocity u. Assume there is no re-polarization.
An electrode in the extra-cellular space at (x=0, y=a, z=0) measures the field potential with respect to infinity. At t=0 **p** is in (x=0, y=0, z=0) .
Find an expression for the electrode signal (v=v(t)).
Help: for a dipole **p** the expression for the potential is (of course) equal to the derivative of the solution found for a single source, i.e.:

$$v(r) = \frac{1}{4\pi\sigma_0}\vec{p}\cdot\vec{r}\,\frac{1}{r^3}$$

c. finally assume there is a re-polarization. Assume that instead of the single dipole p, there now is a pair of dipoles **p** and -**p** at x_0+b and x_0-b respectively, where x0 moves along the axis at constant velocity u. Now find an expression for the electrode signal.

Question 2

A dipole **p** at (0,0,0) is oriented in the x-direction. The potential V(x) caused by this dipole is measured along the line y=0, z=d.
a. Find an expression for V(x) as a function of x, p, d, and σ (the conductivity of the medium)
b. Find an expression for d in terms of Δx, where Δx is the distance between the minimum and maximum of V(x)

Question 3

Two electrodes are placed in a uniform isotropic conducting medium 10 cm from a cell of radius 5um. The two electrodes are 10 cm apart. The two electrodes and the cell form an equilateral triangle. When the cell depolarizes the potential rises by 90mV. $\sigma i/ \sigma 0 = 10$

a. What will be the potential difference between the two electrodes when the cell's orientation is optimal?

b. How many cells would be needed to give a 1 mV potential difference between the electrodes?

Question 4

The small world effect has originally been described by Watts and Strogatz. In their 1998 paper they describe a small world network as one that is in between a regular network and a completely random network. Humphries et al in 2006 introduced the small-world parameter as the ratio of the average clustering coefficient and the average shortest pathlength of the network.

what physical property is described by small-worldness?

Is small-worldness independent of other network properties like connection density or network size?

Does small-worldness completely describe a network?

Can a network with high small-worldness have modules?

Question 5

Damoiseaux et al. have shown that the “default-mode networks” (the resting state networks derived from ongoing fMRI) have similar topology in different subjects.

a. Given that these networks are very similar to the structural networks derived from MRI, motivate why you find (or do not find) Damoiseaux’ results surprising.
b. Given that the connections in our brain are strongly influenced by our past-experience, motivate why you find (or do not find) Damoiseaux’ results surprising.
c. Do these findings mean that all connections are similar between subjects?

Question 6

The power of an EEG recording is strongly influenced by the thickness of the skull.
In this question we would like to get an idea of the strength of this effect. For now we are mostly interested in the magnitude order. We will make a few (over)simplifications:

First: let us assume that resistance of a volume of tissue, where current is flowing in a certain direction is given by:

$$R = \rho \frac{l}{A}$$

where R is the resistance, ρ the resistivity, l the dimension of the tissue in the direction of the current and A the effective area of the tissue (perpendicular to the direction in which the current flows).

Let us further assume that the resistivity of skull tissue is 80x higher than that of skin and brain tissues

Further we assume that we can measure the potential with respect to a reference at infinity; and that the EEG is caused by a single current dipole that causes current to flow radially, i.e. perpendicular to the surfaces between brain and skull and between skull and skin.

a. If the skull is 1cm thick, and the skin is also 1 cm thick, and if the dipolar source is at a depth of 0.5cm beneath the brain surface, how much stronger would the power of the EEG signal right above the dipolar source be when the skull would be removed and replaced by a slab of skin tissue of identical dimensions as the original skull piece?

b. How much would the power of the MEG differ? (does this answer depend on the shape of the head?)

Question 7

When doing intra-cellular recordings the signal picked up the electrode while approaching and finally penetrating the neuron recorded from changes in character. Describe changes that occur when recording from a small spiking neuron, e.g. in the cortex. In your answer describe the changes at low temporal frequencies, and at high temporal frequencies; also consider how the selectivity of the recording changes.

Question 8

In an MEG experiment aimed at localizing the hand area of the primary motor cortex the subject is instructed to alternate between moving the fingers of one hand during 10 seconds and keeping the hand still for 10 seconds.

For the analysis it was decided to use the beamformer method.

a. Would you advise to use a single-state beamformer or a beamformer contrast? And why?
b. Suppose the beamformer contrast was chosen, how would you choose the active and passive time windows of the beamformer?
c. Given your answer to question b, and given that the sample rate in this experiment was 625Hz in which frequency range could the beamformer be constructed?
d. In which frequency range would you expect to see the largest changes in MEG power?
e. Describe a data-driven method to determine the optimal settings for the time window and the frequency range of interest

Question 9

An experimenter injects current into a muscle fiber until the fiber spikes.

The purpose of the experiment is to determine the strength (s) of the injected current as a function of time.

Suppose the current starts abruptly from t=0 seconds, and that the current is stopped abruptly when the intracellular potential reaches a threshold depolarization Vth at t=d seconds.

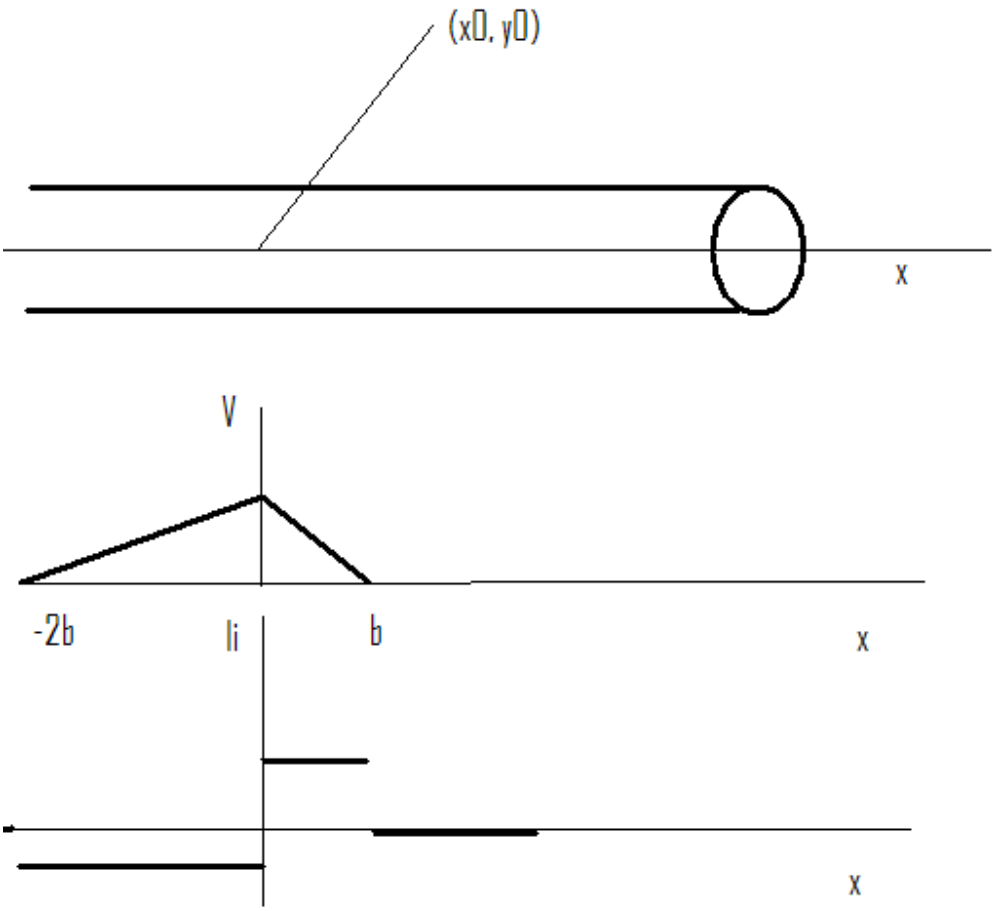
Assume that the fibre is very small and that you may therefore consider that the spatial derivatives for this experiment to be negligible.

a. derive the differential equation that describes how the strength of the injected current s is related to the difference of the membrane potential with respect to the resting membrane potential.
b. Solve this equation, i.e. find an expression for V-Vrest for $t<0$ and for $0<t<d$
c. Find an expression for s in terms of d
d. For which injection duration is the total amount of current injected (i.e. the product of d and s) minimal? (hint: Use the Taylor's series expansion for the exponential function $\exp^{-x}=1 -1/x + (1/2)\ 1/x^2 – (1/6)\ 1/x^3 + \ldots$ to derive your answer.)

Answers to Neurosciences test questions

Question 1.

a. A sketch might help:



We first have to find the currents though the membrane!

Given that the potential varies linearly with x between x=-2b and x=0 and the back from x=0 to x=+b. We know that the current in the dendrite is equal to the conductance on the inside times the spatial derivative of the potential. So:

$$i_i(x) = -\frac{1}{r_i}\frac{\Delta v}{\Delta x} = -\sigma_i \pi a^2 l \frac{\Delta v}{\Delta x}$$

Where l is the length of the element conducting the current.

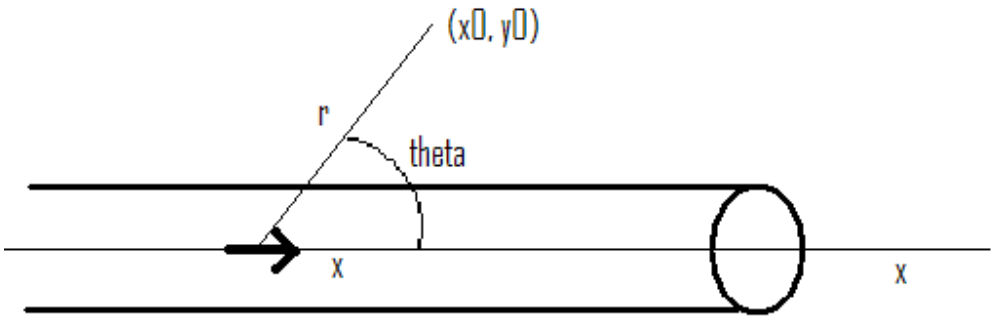
The intracellular current is discontinuous at three positions: x=-2b, x=0, and x=+b. At those positions there must be current through the membrane leaking outside. These are the current sources and sinks that cause the extra-cellular potentials!

So we have a current source at x = -2b and a current source (twice as large) at x=+b and a current sink (strength equal to the sum of the strengths of the two sources) at x=0.

We have been given an expression for the dependence of the extra-cellular potential on the distance and strength of a current source, and the Maxwell equations are linear. So the answer is now long but straightforward:

$$V(x_0, y_0) = \frac{V_0 a^2}{4b} \frac{\sigma_i}{\sigma_o} \left[\frac{1}{\{(x_0 - b)^2 + {y_0}^2\}^{1/2}} + \frac{0.5}{\{(x_0 + 2b)^2 + {y_0}^2\}^{1/2}} - \frac{1.5}{\{{x_0}^2 + {y_0}^2\}^{1/2}} \right]$$

b. Now we have a dipole travelling at constant speed u along the axon.



now:

$$\vec{p} \cdot \vec{r} = pr\cos(\theta) = pr\frac{-x}{r} = -px$$

So we can fill in the equation supplied:

$$v(x) = \frac{1}{4\pi\sigma_o}\frac{-px}{(x^2 + a^2)^{3/2}}$$

And given that x=ut the solution asked is:

$$v(t) = v(x(t)) = \frac{1}{4\pi\sigma_o}\frac{-put}{(u^2t^2 + a^2)^{3/2}}$$

(which in most cases is a bi-phasic function)

c. If there is also a repolarisation pointing in the other direction we get:

$$r^2 = a^2 + (x+b)^2$$
$$\cos(\theta_1) = -(x-b)/r$$
$$\cos(\vartheta_2) = -(x+b)/r$$
$$x_1 = ut - b$$
$$x_2 = ut + b$$

And again the problem is linear so use superposition to get:

$$v(t) = \frac{p}{4\pi\sigma_o}\left[\frac{ut-b}{(a^2 + (ut-b)^2)^{3/2}} - \frac{ut+b}{(a^2 + (ut+b)^2)^{3/2}}\right]$$

(which in most cases is a tri-phasic function).

Question 2.

The answer to a. is obvious after you have seen question 1 :

$$v(x) = \frac{p}{4\pi\sigma}\frac{x}{(x^2+d^2)^{3/2}}$$

For b. we have to find the minimum and maximum of this expression. So let us find where the derivative equals zero:

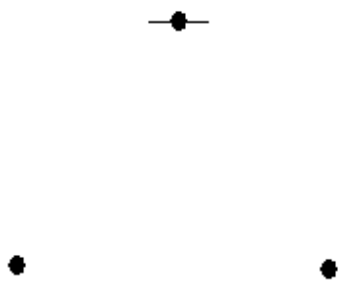
$$\frac{dv}{dx} = 0 = \frac{p}{4\pi\sigma}\left[\frac{1}{(x^2+d^2)^{3/2}} + \frac{-\frac{3}{2}x(-2x)}{(x^2+d^2)^{5/2}}\right] \text{ so:}$$

$$0 = \frac{(x^2+d^2) - \frac{3}{2}2x^2}{(x^2+d^2)^{5/2}} \Rightarrow 2x^2 = d^2 \Rightarrow x = \pm\frac{d}{\sqrt{2}}$$

And thus we find the simple inverse solution here that the depth of the dipole is at $\sqrt{2}$ times the distance between the positions where the potential reaches the maximum and the minimum values

(the inverse is of course only solvable if we knew for sure that v(x) was caused by a single dipole)

Question 3



The largest difference between the potentials at the two electrodes is when p is parallel to the line between the two electrodes.

Given that the configuration is an equilateral triangle both electrodes the see half the dipole (the cosine of 60 degrees is 0.5, that of 150 degrees is -0.5)

(when filling in use SI units here, so the length of each side of the triangle equals 0.1 m, etc.)

Using question 10 (or if you wish the integral formulation from the powerpoint slide)

we get:

$$\Delta V = \frac{1}{4\pi\sigma_o}\left[\frac{pr\cos(\theta_1)}{r^3} - \frac{pr\cos(\theta_2)}{r^3}\right] = \frac{1}{4}\frac{\sigma_i}{\sigma_o}\frac{\pi a^2}{r^2}\Delta V_i = 5.6x10^{-10} Volts$$

If we would want to have an extracellular potential in this situation of 1mV you would need very many aligned cells at the same position:

$$\frac{10^{-3}}{5.6x10^{-10}} = 1.8x10^6 \text{ cells}$$

Answers to questions 4 and 5 should be obvious if you understand the questions.

To question 4: make a clear distinction between the Watts and Strogatz model network (which is a network in the continuum between completely structured and completely random) and the small-world index which can also be calculated for networks of a completely different nature.

Question 6.

Assuming that there are only radial currents is similar to assuming the head is an infinitely large flat and layered conductor.

The first assumption means that all resistivities are homogeneous.

The easiest manner to understand what happens is to introduce the equivalent current pathlength, which is the product of resistance over a path and the physical distance.

Then obviously both problems can be transformed to a problem in half-infinite homogeneous space.

With the skull present the equivalent pathlength between electrode and dipole equals:

1 (skin) + 80 (skull) + 0.5 (brain) = 81.5 units

Without skull the equivalent pathlength equals:

1 (skin) + 1 (skin again) + 0.5 (brain) = 2.5 units.

As the potential due to a dipole drops with distance squared, and as the power is the amplitude squared, the ratio of the powers is:

$$\frac{V_{no\_skull}}{V_{with\_skull}} = (\frac{2.5}{81.5})^4 = 9x10^{-7}$$

For MEG signals in the given geometry something funny happens!

The situation is completely rotation symmetric, which means that there will be no Meg signal at all with or without skull. The ratio would be 0/0 but that does not mean very much.

As soon as the symmetry is broken, the MEG signal will appear, and its amplitude will hardly be affected by the presence or absence of a skull.

Question 7

We are describing what you would record from the cortex.

If you would set your amplifier such that the spiking signal comes through well, then you would see only noise when you are far away from the cells of interest, then when approaching you would see multi-unit activity, that is very many very small spikes from the neurons in the neighbourhood (similar neurons in the cortex are in similar places). The signals would show spikes of a few microVolts sometimes adding up to larger events. When approaching further the signal amplitude would rise to possibly tens of microVolts, and would originate from just a few cells with discernable spike shapes and amplitudes, an extra cellular single unit recording. Finally when the cell is penetrated the spike would be tens of milliVolts large and show a clear refractory period.

If you would set your amplifier to show the low temporal frequencies, then you would be recording the local field potential. The signal would be extremely small when at a large distance, and when approaching the cell closely would grow to possibly a few microVolts, as the PSPs are of course much smaller then the spikes. When penetrating the cell there would be a Dc shift of tens of milliVolts, and the PSPs of a few milliVolts would become apparent. Obviously it is a lot easier to find spiking neurons with your amplifier settings such that spikes are recorded!

Question 8

a. beamformer contrast would work better, you have a clear 2-state experiment
b. windows would be set such that the borders of the task window are omitted. Say from 1 sec after the change in behaviour to 1 sec before the new change. Probably the subjects is presented some cue, either visual or auditory and these responses should not be localized; also the change in behaviour will not coincide exactly with the cue. Allowing the subject some reaction time is a good idea.
c. With 1 sec removed from both sides of the 10 sec windows 2 8 sec epochs remain. So the lowest frequency that can adequately be studied should have 2 (if you are only interested in power changes) or 3 cycles (if you are also interested in phase) in 8 seconds. This corresponds to 2/8 Hz or 3/8 Hz. The highest frequency that can be

studied likewise is determined by the Nyquist frequency which is 0.5 * the smapling frequency or 312.5 Hz

d. We expect the strongest changes somewere in the beta band
e. Construct the spectrogram and determine the frequency band and time windows within the 2 states where the power changes are most outspoken.

Question 9

a.

$$Rs(t) = RC\frac{dV}{dt} + V$$

Where R is the total membrane resistance, C the total membrane Capacitance and V is the membrane potential minus the resting potential

b.

solving for t<0 s(t) = 0 and thus V=0

solving for 0<t<d s(t)=S and thus

$$V(t) = RS\left\{1 - e^{-t/RC}\right\}$$

Solving for t>d is not possible, that is governed by the spiking dynamics and can only be done numerically

c.

at t=d the fibre spikes, so V(d) = Vth, so:

$$Vth = RS\left\{1 - e^{-d/RC}\right\}$$

And thus:

$$S = \frac{Vth}{R\left\{1 - e^{-d/RC}\right\}}$$

d.

we will have to take the derivative of dS to d and set that to 0.

$$\frac{d(dS)}{dd} = \frac{d}{dd}\left\{\frac{dVth}{R\left\{1 - e^{-d/RC}\right\}}\right\} = 0$$

From which we will have to find d!

Obviously we can immediately remove Vth and the factor 1/R and we will find:

$$1 - e^{-d/RC} - \frac{d}{RC}e^{-d/RC} = 0$$

Using x=d/RC and the Taylor's series expansion

$$e^{-x} = 1 - x + \frac{x^2}{2} - \frac{x^3}{6} + \ldots$$

We get:

$$\left(\frac{1}{2}x^2 - \frac{1}{6}x^3\right) = 0$$

or $x = \frac{3}{2} \quad \Rightarrow d = \frac{3RC}{2}$